QUICK NOTES - INVESTMENTS

Overview

- 3 aspects of risky decision making:
  - Prices
  - Probabilities
  - Preferences
- Risk vs. Uncertainty
  - Uncertainty is different from risk in that risk is mathematically quantifiable and uncertainty is not
  - We want to look at the tradeoff between risk and expected return
- Risks in the Long Run
  - Does S&P 500 dominate T-Bills?
    - Not enough data since $R_1(20)$ and $R_2(20)$ highly correlated
    - Early withdrawal
    - The actual time period used is an issue here
    - Data biases → rich successful country; world financial markets do not have this property; must include losers
    - If you believe that the gap between stocks and bills is a risk premium, then everyone would invest in the S&P and you would drive down the return of the S&P
    - Taking a bet a number of times is not necessarily less risky than taking a single bet
    - Tracking error between stocks and bills increases with T
- Risk definitions:
  - Risk aversion: Prefer a certain outcome to an uncertain outcome with an equivalent expected value
  - Risk seeking: Prefer an uncertain outcome with an equivalent expected value to a certain outcome
  - Risk neutral: Indifferent between a certain outcome and an uncertain outcome with an equivalent expected value
- Investment problem

Functional Perspective

- Flow model of the economy
  - Product markets, labor markets, households, nonfinancial corporations, capital markets and financial intermediaries
- Functions stable, not institutions → Innovations in financial technology
- Six functions of the financial system
  - Payments system
  - Pooling of funds for large-scale investment
  - Resource transfer
  - Risk management
  - Information to coordinate decision-making
  - Resolution of agency problems
- Three problems of finance
  - What to invest in?
  - When to invest in it?
  - How to pay for it?

Statistical Inference in Finance

- Risk vs. Uncertainty
- Random Variables
  - A rule that assigns a numerical value to each possible outcome of a probabilistic experiment

  - A model of the unknown that has certain characteristics, including a distribution and statistics
- Classical statistical inference
  - Data X (random variable until observed → x)
  - Probability law $P(X;\theta)$ (The "Model")
  - θ fixed but unknown, such that $\theta \in \Theta$; θ is a population parameter
  - From X, obtain "best guess" for θ → θ-hat(X) = f(X) ➔ close to θ
  - θ-hat(X) is an estimator; θ-hat(x) is an estimate (either right or wrong)
  - Choose θ-hat(X) as a "best" procedure → choose f(X) as a "best" procedure
  - Do not choose θ-hat(x)
  - "Best"
    - Best Linear Unbiased Estimator (BLUE) → minimum variance (error around estimator)
    - Unbiasedness not sufficient → counter-example of binary variable
    - Consistency more relevant for finance
      - $\nabla \in > 0$, $\nabla\ \theta \in \Theta$, → lim (n→∞) Pr(|$\theta$-hat - θ| > $\in$) = 0
      - With more data, the estimator becomes more centered around the true parameter
      - Example: Mean estimation
      - Counterexample: $Y_k = Z + X_k$ → Y-bar = (1/n)Sum(Y_k) is not consistent
      - Consistency may fail due to dependence
    - Stationarity is a type of allowable dependence:
      - $Pr(X(t_1), X(t_2), \ldots, X(t_n)) = Pr(X(t_1+\tau), X(t_2+\tau), \ldots, X(t_n+\tau))$
      - $E(X(t)) = \mu$
      - $Var(X(t)) = \sigma^2$
      - $Cov(X(t_1),X(t_2)) = \gamma(t_2-t_1)$
      - $Cov(X(t_1),X(t_2)) = \rho(t_2-t_1)$
      - $\rho_k = Cov(X_t, X_{t-k})/Var(X_t)$ ← the k-th order autocorrelation
      - γ_k is the k-th order autocovariance
      - For most time series, $\rho_k \to 0$
    - Under stationarity, the law of large numbers states that estimators get closer to the true parameters as n → ∞
- Simple tests of significance
  - Can compute test statistics by normalizing
  - Example:
    - X(1), …, X(n) IID $N(\mu, \sigma^2)$
    - X-bar = (1/n) x Sum (X) ~ $N(\mu, \sigma^2/n)$
    - Pr(X-bar > c) = Pr(X-bar - μ > c - μ)
    - Normalize by dividing by σ/sqrt(n) → lookup in standard normal tables
    - Central Limit Theorem:
      - With a large enough sample, even if the original distribution of X's is not normal, the distribution of X-bar is asymptotically normal $N(\mu, \sigma^2)$
  - Example: X(1), …, X(n) IID; $E(X_k) = \mu$, $E[(X_k - \mu)^2] = \sigma^2$, $E[(X_k - \mu)^4] = \delta$
    - CLT → X-bar → $N(\mu, \sigma^2/n)$
    - CLT → σ^2-hat → $N(\sigma^2, (\delta-\sigma^4)/n)$
    - CLT → ρ_k-hat → N(0, 1/n)
- Estimating means, covariances and correlations (see handwritten notes)

  Population parameters:

  $E(X) = \mu_X = \sum_j p_j X_j$

  $Var(X) = \sigma_X^2 = \sum_j pj(Xj - \mu_j)^2$

$Cov(X,Y) = \sigma_{XY} = \sum_{ij} \; p_i p_j \; (Xj - \mu_X)^2 (Y_i - \mu_Y)^2$

$Corr(X,Y) = \rho_{XY} = Cov(X,Y)/\sigma_X \sigma_Y$

Sample parameters:

Estimate of $\mu_X = (1/n) \sum_j X_j$

Estimate of $Var(X) = (n-1)^{-1} \sum_j (Xj – Est(\mu_j))^2$

Estimate of $Cov(X,Y) = (n-1)^{-1} \sum_j (Xj – Est(\mu_X)) (Yj – Est(\mu_Y))$

Estimate of Corr(X,Y) = Estimate of Cov(X,Y) / [Estimate of Var(X) x Estimate of Var(Y)]

- Linear regression
  - β-hat = Cov-hat (X_t,Y_t) / Var-hat (X_t)
  - α-hat = Y-bar - β-hat x X-bar
  - Goodness of fit R^2 –hat = 1 – (Var-hat(ε_t) / Var –hat (Y_t))
  - Sampling theory

Household Preferences:

- Preferences under certainty
  - Prefer higher return with lower variance
  - Diminishing marginal utility and convex indifference curves
- Fisher separation
  - Investors are better off with capital markets in which you can separate ownership and control
  - Maximizing NPV of wealth is correct for all investors
- Preferences under uncertainty
- Expected utility theory
- Constructing preferences

Lifecycle Investing and Risks in the Long Run

- A dynamic problem, age matters
- Stochastic dynamic programming
- Definition of risk unclear
- Preferences matter
- Risks do not necessarily “average out”
  - Taking a number of bets may be riskier than taking a single bet
  - Check out the degree of correlation between the R_t’s
- Put-Option analysis
  - With probability 1, the stock market will do worse than T-Bills eventually
  - Risks don’t cancel out in the long run → they increase
  - Insurance costs more over time
  - Using Black-Scholes assumptions

Price Formation and Market Efficiency

- Pricing stocks and bonds under certainty

  - Dividend Pricing Model P = Div/R
  - Assuming that there is a constant stream of dividends (a perpetuity)
  - If growing at rate g then P = Div / (R-g)
- Market Efficiency:
  - Weak Form: Prices reflect all available information in past prices (technical analysis impossible)
  - Semi-strong Form: Prices reflect all publicly available information (cannot make money from earnings)
  - Strong Form: Prices reflect all publicly available information and all private information (insider traders cannot make money)
- Early tests of efficiency concluded that markets are efficient:
  - Weak → Random-walk hypothesis
  - Semi-strong → Event studies
  - Strong → Insider transactions
- Existence of pricing anomalies:
  - January effect
  - Valueline puzzle
  - Weekend effect
  - "Data-snooping" biases
- Recent tests of efficiency
  - Tests of earnings forecasts found them to be biased and irrational
  - Tests of efficiency are joint tests
- Rational expectations vs. hog-cycle
  - Convergence not necessary
- Trading process is complex
- Information revealed through prices
- Classical efficiency markets hypothesis
- Modern efficient markets hypothesis
  - Rents accrue to financial technology
  - Markets are not frictionless
  - Think of efficiency as competitiveness and think of markets as an ecology
  - There is a reason for making money, otherwise it won't be consistent

The Random Walk Hypothesis

- The martingale:
  - $E[P_t|\Omega_{t-1}] = P_{t-1} \rightarrow P_t = P_{t-1} + \varepsilon_t$ such that $E[\varepsilon_t|\Omega_{t-1}] = 0$
- Three versions of the random walk
  - Weak: Uncorrelated Increments (RW1)

    Martingale with $E[\varepsilon_t] = 0$ and $E[\varepsilon_t\varepsilon_{t-k}] = 0$ for $k > 0$

  - Semi-Strong: Independent Increments (RW2)

    Martingale with $E[\varepsilon_t] = 0$ and $E[f(\varepsilon_t\text{-fbar})g(\varepsilon_{t-k} – \text{gbar})] = 0$ for $k > 0$

  - Strong: IID Increments (RW3)

    Martingale with ε_t IID

- Properties of the Random Walk
  - Unforecastable returns
  - Uncorrelated increments
  - Linearity of variances
- Early tests
  - Cowles and Jones rejected RW hypothesis incorrectly

    - What they actually rejected was the zero mean hypothesis
    - Did not correct for drift
    - Once drift is corrected for, there are no more rejections
  - Other tests failed to reject the RW hypothesis
- Variance ratio test
  - Tests the linearity of variance property of the RW hypothesis

$VR(q) = Var[R_t(q)] / (q\ Var[R_t])$

$Var[R_t(q)] = Var[R_t + R_{t-1} + \ldots + R_{t-q+1}]$

Null hypothesis: Under the random walk VR(q) should be equal to 1

Generally VR(q) can be shown to be

$VR(q) = 1 + 2 \sum (k=1 \text{ to } q-1)\ (1 - k/q)\ \rho_k$

Generally rejects the null hypothesis for all q

- Empirical evidence
  - Large positive autocorrelation for indexes
  - Stronger rejection for small firms
  - Cannot reject for individual stocks whose average autocorrelation is weakly negative

Corporate Preferences for Risk and Expected Return

- MM propositions and risk
  - The value of the firm is independent of the firm's capital structure → can undo leverage as an investor by buying the firm's bonds
  - Expected rates of return on E and D are not constant, even though the expected return on assets is constant → individuals can diversify their diversifiable risk so therefore risk management is unnecessary and undesirable
  - Corporate preferences → maximize E and of course risk management plays a role
- Violations of MM
  - Taxes → induces convexities in after-tax cash flow → risk has nonlinear effects
  - Transactions costs
  - Unlimited borrowing and lending
  - Fixed investment policy
- Risk can affect shareholder wealth
  - Stockholder-bondholder conflict
- Investment incentives
- Costs of financial distress
- Taxes (induces convexities)

The Risk Management Process

- Identify risk exposures
- Evaluate Value-at-Risk (VaR)
  - Identify X
  - Identify functional dependence between ΔV and X (e.g. linear factor model)
  - Given X_t, simulate multiple ΔV_t's by simulating different error terms
  - Plot histogram and take out VaR from distribution
- Target risks to hedge
- Select hedging vehicles
- ***Evaluate post-hedge VaR***

Calculating VaR

- *P(ΔV|X)*, conditional on information X
- *P(ΔV)*, requires P(X)
- Procedure for simulating $\{\Delta V_t(1), \ldots, \Delta V_t(N)\}$
- Modeling ε_t crucial
  - There are different times of processes ARIMA, stochastic volatility
- Notes
  - Risks that can be hedged most easily may not be the most important risks
- Shortcomings of VaR
  - Generic → not geared toward a particular application
  - Static → "snapshot" and it ignores the joint distribution of multiple-day risks
  - Parametric → relies on distributional assumption
  - Focused on outliers → not accurate estimation if there are only a few observations
  - Purely statistical → ignores economic cause and effect
- Risk management for hedge funds
  - Data issues and survivorship bias
  - Dynamic risk management
  - Correlation and risk adjustments
  - Risk and performance attribution
  - Psychology of risk preferences
  - Position transparency ≠ Risk transparency
  - Beware of phase-locking correlation

Financial Intermediation

- Growing importance of intermediation
- Economics of intermediation
  - Coase's "Nature of the Firm" → reduction of transactions costs and the facilitation of interaction between agents
  - Big opportunities for market-making firms
- Four functions of intermediation
  - Price discovery
  - Liquidity and immediacy
  - Matching and searching
  - Guaranteeing and monitoring → reducing impact of asymmetric information

Asset Management

- Industry overview
- Objectives of the portfolio
- Stock selection
- Asset allocation
  - Mutual fund separation theorem: Choosing among many securities can boil down to choosing CAPM between risk-free rate and the efficient frontier → where is the point of tangency between the investor's indifference curve and the security market line
- Option pricing and asset allocation and perfect market-timing
  - 2 assets: riskless and market
  - $Z_{ft} \equiv 1 + R_{ft}$; $Z_{mt} \equiv 1 + R_{mt}$
  - Invest A_t in stocks or bonds at time t
  - Pay management fee F_t
  - Gross investment $I_t = A_t + F_t$
  - Value of investment at time t+1: $V_{t+1} = \text{Max}[A_t Z_{mt}, A_t Z_{ft}] = A_t \text{ Max}[Z_{mt}, Z_{ft}]$
  - This is equivalent to earning $Z_{mt} + \text{Max}[0, Z_{ft} - Z_{mt}]$

- This is the same as investing in the stock and having a put on the stock market struck at the risk free return
- This is also the same as investing in a call on the stock market
- The rate of return is "fair" or the market rate, predicated on Black-Scholes model
  - Set the fee so that the investor is indifferent between doing business in the market or with the manager

- Marketing
- Performance evaluation

Testing the CAPM and the APT

- Type I and Type II errors
  - Type I error: Rejecting when the null hypothesis is true
  - Type II error: Not rejecting when the null hypothesis is false
  - Size ≡ Probability of a Type I error (e.g. 5% for a 5% confidence interval)
  - Power ≡ 1 – Probability of a Type II error (depends on alternative hypothesis)
  - There is a tradeoff between size and power
- Mutual fund separation theorem: Investor indifferent between choosing between the whole universe of assets and two assets: the risk-free asset and a mutual fund
- CAPM
  - A one-period equilibrium model that starts with the canonical investment problems → mean-variance preferences by the assumption of constant relative risk aversion and has equilibrium imposed upon it
  - All investors behave similarly
  - Pick non-negative weights → matrix of optimal weights proportional to tangency portfolio
  - All assets held and supply = demand → M must be the market portfolio since all individuals are identical
- Implications of CAPM (and its market clearing assumption):
  - All investors choose the same optimal risky portfolio and this is the market portfolio
  - Market portfolio is on frontier
  - Market portfolio is the tangency portfolio
  - Risk premium on individual assets is β (covariance of asset with market divided by variance of the market)
  - Linearity of risk and expected returns
  - **Sharpe Ratio = $(\mu_m – R_f)/\sigma_m$ as performance measure**
  - The capital market line is in μ-σ space; the security market line is in μ-β space
  - Under-priced if above the security market line; over-priced if below the security market line
- Black's Zero-Beta CAPM
  - Riskless asset does not exist
  - Redo the investment problem
  - Choose weights to maximize value of the portfolio (a function of the weights, expected returns and the variance-covariance matrix subject to the constraint that all weights sum up to one
  - All investors turn out to be indifferent between choosing 2 assets and choosing among the whole universe of assets (an example of a mutual fund separation theorem)
  - We can prove that there always exists a zero-beta portfolio with always positive weights and that this zero-beta portfolio is unique

$$\mu_I - \mu_z = \beta_I(\mu_m - \mu_z)$$

$$\beta_I \equiv Cov(R_i, R_m) / Var(R_m)$$

  - The market portfolio lies on the efficient frontier at the point of tangency with the capital market line

- Any 2 portfolios on the efficient frontier is sufficient to generate the entire curve using linear combinations of these 2 portfolios
- All of the risk in the zero-beta portfolio is diversifiable → $\beta_z = 0$
- If there does exist a risk-free asset, it must be true that the expected return on the zero-beta asset = R_f

- Arbitrage Pricing Theory
  - Assume a linear return generating process

    $R_i = \mu_I + \beta_i\Lambda + \varepsilon_I$, ∇ I = 1,…, m

    $E[\Lambda] = E[\varepsilon] = 0$

    Consider any arbitrage portfolio ω_a such that the weights sum to zero (an arbitrage portfolio) and the return is determined by the linear return generating process above, assuming that the product of the weights and the errors is zero, then choose weights so that the product of the weights and the β matrix are zero → by no arbitrage therefore the product of the weights and the expected returns vector is itself zero

    μ lies in the plane spanned by ι and β → $\mu = \gamma_0\iota + \gamma_1\beta$

    Consider the riskless asset R_f: $\gamma_0 = R_f$

    Consider the Portfolio with unit Λ-risk → $\omega_m'\beta = 1$

    $\omega_m'\mu = R_f + \gamma_1\omega_m'\beta = R_f + \gamma_1$ → $\gamma_1 = \mu_m - R_f$

    $E[R_i] = R_f + \beta_I(E[R_m]-R_f)$ ← Same relation as CAPM without equilibrium or preferences

    Can obtain CAPM result based purely on the no-arbitrage condition

- Multi-factor generalization

    $R_i = \alpha_I + \beta_{1i}\Lambda_1 + \ldots \beta_{Ki}\Lambda_K + \varepsilon_I$

    $E[\varepsilon_I] = 0$

    $E[\varepsilon_i\varepsilon_j] = 0$, for all $i \neq j$

    In matrix notation:

    $$\mathbf{R = \alpha + \beta\Lambda + \varepsilon}$$

    $$E[\varepsilon\varepsilon'] = \mathbf{Diag}(\sigma_1^2, \ldots, \sigma_n^2)$$

    $$E[R_i] = R_f + \beta_{1i}(E[R_{\lambda 1}] - R_f) + \ldots + \beta_{Ki}(E[R_{\lambda K}] - R_f)$$

- Early tests of CAPM exhibit some rejections
  - Performance evaluation models for mutual funds → generally conclude that CAPM holds
- Recent tests exhibit many rejections
  - Multivariate testing approach (both time-series and cross-section)
  - Test the null hypothesis that in the regression of individual stock returns on the market that the constant term is equal to $R_f(1-\beta_I)$ without specifying an alternative hypothesis
  - In the Black zero-beta CAPM, estimate the zero beta expected return instead
  - Reject the Black zero beta CAPM and reject the Sharpe-Lintner CAPM

- Problems
  - Asymptotic approximations break down → see Monte Carlo testing of results
  - Size – MC simulations for only 5 portfolios
  - Power – irrelevant alternatives
- Fama and French (1992)
  - Include the $\log(Size_i)$ as a regressor
  - β_I seems to have wrong sign and it is statistically insignificant
  - In a separate paper they suggest $\log(Book_i/Size_i)$
    - Value Stocks have high Book/Market
    - Growth Stocks have low Book/Market
    - Loughran argued against value beating growth for 3 reasons:
      - Value funds don't necessarily beat growth funds in data
      - S&P 500 tilted towards large growth, difficult to beat in practice with small value stocks
      - Repeating the Fama and French analysis without NASDAQ and without January returns makes Book/Market and Size irrelevant and β positive again
  - 3 possible explanations
      - Market inefficiency
      - Risk factors
      - Data-snooping biases → difficult to distinguish effects (identification problem)
- Problems with distributional assumptions
- An uncertainty principle, lack of power
  - Null hypothesis: $\alpha = 0$ in regression

    $R_{it} - R_{mt} = \alpha_I + \beta_I X_{mt} + \varepsilon_{it}$

    $X_{mt} \equiv R_{mt} - R_{ft}$

    H_o: Estimated $\alpha_I \sim N(0,V_a)$

    Uncertainty arises when testing to see if α is sufficiently small given the small order of magnitude of estimate V_a
- BARRA model
  - Variance-covariance matrix has a dimensionality problem if n > T (singularity)
  - BARRA model uses a K-factor model to reduce dimensionality

    $\mathbf{R} - \iota \mathbf{R_f} = \mathbf{B}\Lambda + \varepsilon$

    $E[\varepsilon\varepsilon'] = D = \mathrm{Diag}(\sigma_1^2,\ldots)$

    $\sum = \mathbf{B\Omega B' + D}$ where Ω is KxK not nxn

    $\mathrm{Cov}(R_i,R_j) = \beta i \beta_j \sigma_\lambda^2$

    $R_i = \alpha_I + \beta_i\Lambda + \varepsilon_I\ \nabla I$

Fixed Income Securities

- Arbitrage relations
- Estimating the term structure
- Duration and convexity
- Corporate bonds and default risk
- Models of the term structure

Finance and Insurance

- Risk/reward tradeoff in insurance
- Similar goals and methods
- Innovation is critical
- Opportunities are huge (intellectual arbitrage)

Hedge Funds and Proprietary Trading

- Large positive index autocorrelation
  - Index autocorrelation is the result of own-autocorrelations and cross-autocorrelations
  - Consider a market $R_t \equiv [R_{at}\ R_{bt}]'$ that is stationary with expected mean vector μ and auto-covariance matrix Γ_k describing the co-variance of R_t across time by time-difference k periods → auto-correlation matrix Υ_k
  - Consider an equal weighted index $R_{mt} = 0.5\ R_{at} + 0.5\ R_{bt}$
  - $Cov(R_{mt},R_{mt+1}) = 0.25\ Cov(R_{at},R_{at+1}) + 0.25\ Cov(R_{bt},R_{bt+1}) + 0.25\ Cov(R_{at},R_{bt+1}) + 0.25\ Cov(R_{bt},R_{at+1})$ = Own-Effects + Cross-Effects
  - Own effects are small and negative
  - Cross effects are large and positive
- Lead/lag patterns
  - Consider a factor model in which R_{at} is dependent on factor vector Λ_{t-1} and R_{bt} is dependent on factor vector Λ_{t-2}
  - Ordering is critical
  - Asymmetric cross effects
  - Classification is critical
  - Stability over time
  - Economics → lagged adjustment of information, learning behavior, nonsynchronous trading, etc.
  - Trading → Profitable dynamic trading strategies exist
- Contrarian trading strategies exploit reversals
  - Can imagine one in which you took the opposite position in an asset to the excess of its return over the market k periods ago → e.g. if excess profit k periods ago, short it today, etc.
- Inadvertently exploits lead/lag effects
  - Selling stock b after a positive return on stock a can lead to an inadvertent profit, not intended to take advantage of lead/lag effects
- Profitability substantial
- Magnitude of trading costs critical

Stock Selection

- Exploiting mispricings optimally

  Most stocks priced optimally → $E[R_i] = R_f + \beta_I(E[R_m] - R_f)$

  Some stocks mispriced → $E[R_j] = \alpha_j + R_f + \beta_j(E[R_m] - R_f)$

- Exploiting this mispricing means using active trading

  - Let A denote the Active portfolio
  - Let M denote the Passive Portfolio

$$E[R_a] = \alpha_a + R_f + \beta_a(E[R_m] - R_f)$$

- The problem: how to combine A and M?

  $E[R_p] = \omega_p E[R_a] + (1-\omega_p)E[R_m]$ ← Choose ω_p to yield tangency portfolio

  The tangency between the capital market line and the efficient frontier

- How good is P?

  $[(E[R_p] - R_f)/\sigma_p]^2 = [(E[R_m] - R_f)/\sigma_m]^2 + [\alpha_a / \sigma(\varepsilon_a)]^2$

  $S_p^2 = S_m^2 + [\alpha_a / \sigma(\varepsilon_a)]^2$

  $(\text{Sharpe Ratio for P})^2 = (\text{Sharpe Ratio for Market})^2 + (\text{Appraisal Ratio})^2$

  Select weights for A to maximize the Appraisal Ratio → enhanced indexing

  Portable α: Take a strategy from one market and apply it to another, using derivatives (e.g. total return swap) to transform α from one market to another

Nonlinearities in Financial Markets

- Plausible motivation for nonlinearities
- Many techniques available, estimation more difficult
- Nonparametric techniques involve tradeoffs
- Less restrictive parametric assumptions
- More restrictive in other ways
- Not of proven value yet, but growing research area